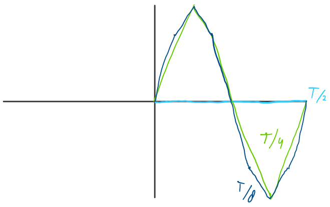
Exercises chapter 2

Wednesday, 13 November 2019 20:30

1) $f_{\frac{T}{2}}\cancel{[k]} = \{0, 0, 0\}$

$f_{\frac{T}{4}} = \{0, 1, 0, -1, 0\}$

$f_{\frac{T}{8}} = \{0, \frac{1}{2}\sqrt{2}, 1, \frac{1}{2}\sqrt{2}, 0, -\frac{1}{2}\sqrt{2}, -1, -\frac{1}{2}\sqrt{2}, 0\}$



For $\frac{T}{2}$, the signal contains only zeroes

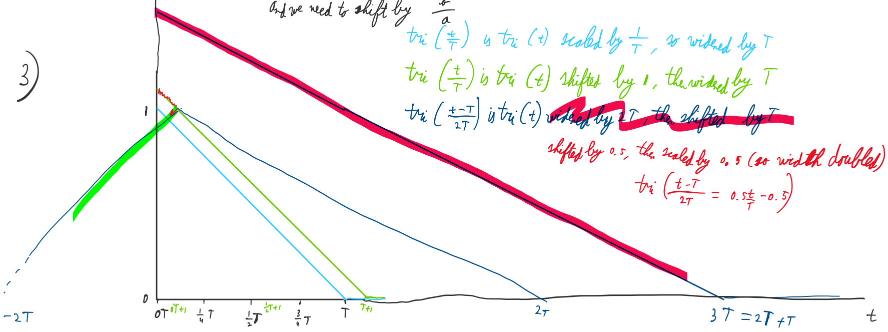
idea: replace t in-place by an 'action' $t-b$ or at

$f(at-b) = f\left(a\left(t-\frac{b}{a}\right)\right)$

2) yes, but in that case, we need to scale by a

and we need to shift by $\frac{b}{a}$

3)



$\text{tri}\left(\frac{t}{T}\right)$ is $\text{tri}(t)$ scaled by $\frac{1}{T}$, so widened by T

$\text{tri}\left(\frac{t}{T}\right)$ is $\text{tri}(t)$ shifted by 1, then widened by T

$\text{tri}\left(\frac{t-T}{2T}\right)$ is $\text{tri}(t)$ ~~widened by 2T, then shifted by T~~

shifted by 0.5, then scaled by 0.5 (so width doubled)

$\text{tri}\left(\frac{t-T}{2T} = 0.5\frac{t}{T} - 0.5\right)$

4) Let $f(t)$ be even; let $g(t)$ be odd, then define

$h(t) = f(t)g(t) = f(-t) \cdot -g(-t) = -f(-t)g(-t) = -h(-t)$, so $h(t)$, the product of an even and an odd function, is odd

5a) not necessary, but important for the sinh function!

$f(t) = e^{-vt} = \frac{1}{2}\left(e^{-vt} + e^{vt}\right) + \frac{1}{2}\left(e^{-vt} - e^{vt}\right) = \cosh(-vt) + \sinh(-vt)$

even odd

5b) $= \frac{1+t}{(1-t)^2}$

$f(t) = \frac{1}{1-t} = \frac{1}{2}\left(\frac{1}{1-t} + \frac{1}{1+t}\right) + \frac{1}{2}\left(\frac{1}{1-t} - \frac{1}{1+t}\right) = \frac{1}{2}\left(\frac{1+t+1-t}{(1-t)(1+t)}\right) + \frac{1}{2}\left(\frac{1+t-1+t}{(1-t)(1+t)}\right) = \frac{1}{2}\cdot\frac{2}{1-t^2} + \frac{1}{2}\cdot\frac{2t}{1-t^2} = \frac{1}{1-t^2} + \frac{t}{1-t^2}$

even odd

6.1) $\int_{-a}^{a} f_+(t)\,dt = \int_{-a}^{0} f_+(t)\,dt + \int_{0}^{a} f_+(t)\,dt = -\int_{-a}^{0} f_+(-t) + \int_{0}^{a} f_+(t)\,dt = \int_{0}^{0} f_+(-t) + \int_{0}^{a} f_+(t)\,dt$

define $\tau = -t$

then $\int_{-a}^{0} f_\pm(t)\,dt = -\int_{a}^{0} f_\pm(-\tau)\,d\tau = \int_{0}^{a} f_\pm(-\tau)\,d\tau = \pm\int_{0}^{a} f_\pm(\tau)\,d\tau$

7) x^4 is even and

$\sin(x)$ is odd, so

$x^4\sin(x)$ is odd, so it does not contribute to the integral symmetric about 0

so, $\int_{-\pi}^{\pi} 1 + x^4\sin(x)\,dx = \int_{-\pi}^{\pi} 1\,dx = [x]_{-\pi}^{\pi} = \pi - -\pi = 2\pi$

8) $g\left(t + \frac{T}{a}\right) = f\left(a\left(t + \frac{T}{a}\right)\right) = f(at + T) = f(at) = g(t)$

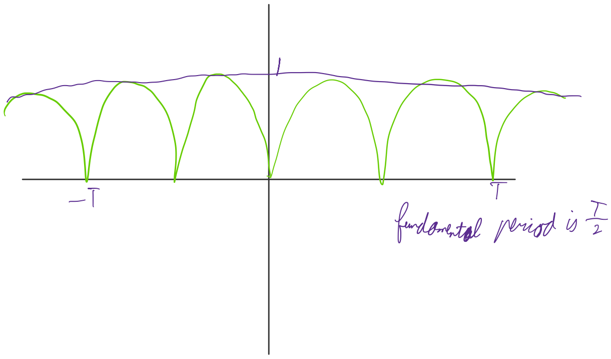
9) The period of $f(t)$ is T

9_2) $f\left(t-\frac{T}{2}\right) = \max\left(0, \sin\left(2\pi\left(t-\frac{T}{2}\right)/T\right)\right) = \max\left(0, \sin\left(2\pi t/T - \pi\right)\right)$, which has period T

9_3) $g(t) = f(t) + f\left(t-\frac{T}{2}\right)$

$T_g = mT_1 + nT_2$

$\frac{T_1}{T_2} = \frac{T}{T} = 1 = \frac{1}{1} = \frac{n}{m}$, so $T_g = T$



fundamental period is $\frac{T}{2}$

10_1) $\tan(t)$ is periodic with fundamental period $T_0 = \pi$ ✓

10_2) is periodic, ~~$T_0 = 3\pi$~~ periods are π and $\frac{\pi}{3}$, so $T_0 = \pi$ X

10_3) not periodic ratio is $\sqrt{2} \notin \mathbb{Q}$ ✓

10_4) periodic, $T_0 = 2\pi$ $\sin\sin(t+2\pi) = \sin\sin t$, $T_0 = 2\pi$ ✓

10_5) not periodic → "2T_0 would not be a period if T_0 would be one"

11_1) not periodic $\left(\frac{1}{2\pi} \notin \mathbb{Q}\right)$

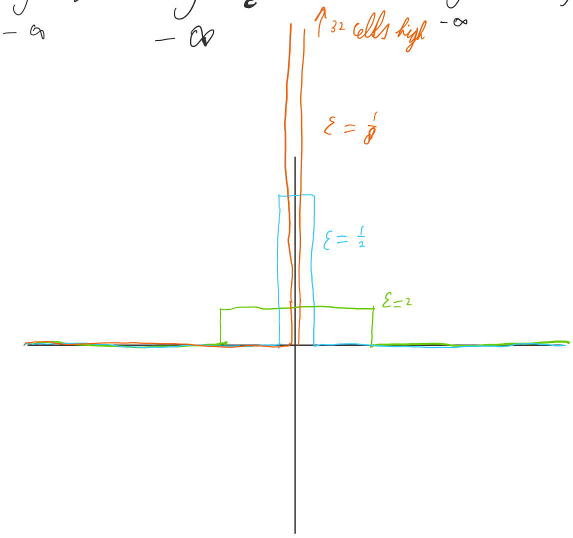
11_2) periodic: $N_0 = 4$

11_3) periodic: $N_0 = 4$

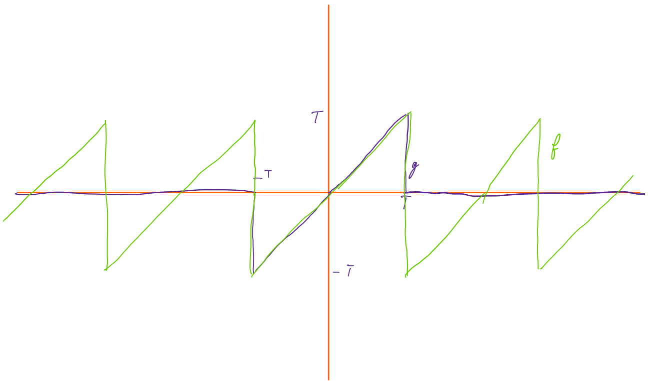
12) $s(t) = \sin(2\pi t/T)\,\mathrm{rect}\left(\frac{t-T}{2T}\right)$ period = T

13_1) $\int_{-\infty}^{\infty} D(\varepsilon, t) = \int_{-\infty}^{\infty} \frac{1}{\varepsilon}\mathrm{rect}\left(\frac{t}{\varepsilon}\right)dt = \frac{1}{\varepsilon}\int_{-\infty}^{\infty}\mathrm{rect}\left(\frac{t}{\varepsilon}\right)dt = \frac{1}{\varepsilon}\cdot\int_{-\frac{\varepsilon}{2}}^{\frac{\varepsilon}{2}} 1\,dt = \frac{1}{\varepsilon}\left[t\right]_{-\frac{\varepsilon}{2}}^{\frac{\varepsilon}{2}} = \frac{1}{\varepsilon}\left(\frac{\varepsilon}{2}+\frac{\varepsilon}{2}\right) = \frac{\varepsilon}{\varepsilon} = 1$

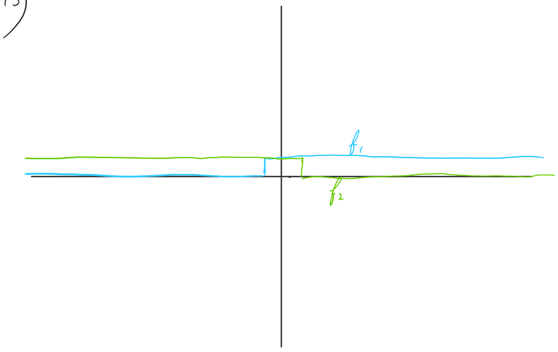
13_2)



14)



15)



16) $\text{rect}(t) = u(t+\tfrac{1}{2}) \cdot u(-(t-\tfrac{1}{2})) =$ ✓
$u(t+\tfrac{1}{2}) \cdot u(\tfrac{1}{2}-t)$

$\text{rect}(t) = u(t+\tfrac{1}{2}) - u(t-\tfrac{1}{2})$ $= 1 \Leftrightarrow |t| < \tfrac{1}{2}$ ✓

shift ½ left ↑ shift ½ right

2.17_1) $\int_{-\infty}^{\infty} f(t-a)\,\delta(t)\,dt = f(-a)$

2.17_2) $\int_{-\infty}^{\infty} f(t-a)\,\delta(t-t_0)\,dt = f(t_0 - a)$

← out of bounds $t = 1$ ↑

2.17_3) $\int_{-\infty}^{0} f(t)\,\delta(t-1)\,dt = 0$ ✓

$\tau = at - t_0 \rightarrow at = \tau + t_0 \rightarrow t = \frac{\tau}{a} + \frac{t_0}{a}$
$d\tau = a\,dt$

2.18) $\int_{-\infty}^{\infty} f(t)\,\delta(at - t_0)\,dt =$

$\frac{1}{|a|}\int_{-\infty}^{\infty} f\left(\frac{\tau + t_0}{a}\right)\delta(\tau)\,d\tau = \frac{1}{|a|} f\left(\frac{t_0}{a}\right)$

negative a : boundaries change as well → cancels out

19_1) $\int_{-\infty}^{\infty} f(t-a)\,\delta(2t)\,dt = \frac{1}{2} f\left(\frac{0}{2}\right) = \frac{1}{2} f(0-a)$

19_2) $\int_{-\infty}^{\infty} f(t-a)\,\delta(-t)\,dt = -f(0-a) = -f(-a)$

~~19_3) $\int_{-\infty}^{\infty} (\delta(t/2) - \delta(t) - \delta(-t))\,dt = \int_{-\infty}^{\infty} (\delta(t/2) - 2\delta(t))\,dt$~~
~~$= \int_{-\infty}^{\infty} \left(\delta(t/2) - \delta\left(\frac{t}{2}\right)\right)dt = \int_{-\infty}^{\infty} 0\,dt = 0$~~

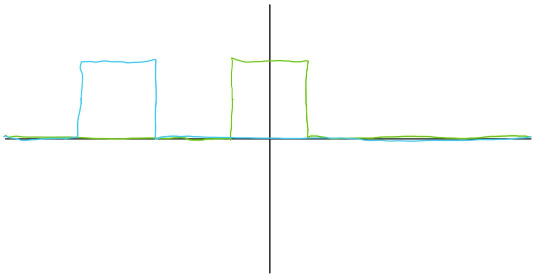
19_3) $\int_{-\infty}^{\infty} (\delta(t/2) - \delta(t) - \delta(-t))\,dt = \int_{-\infty}^{\infty} (2\delta(t) - \delta(t) - \delta(-t))\,dt = 2 - 1 - 1 = 0$

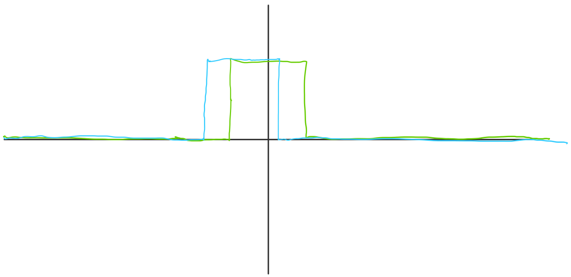
$$f(t) * g(t) = \int_{-\infty}^{\infty} f(\tau)\, g(t-\tau)\, d\tau$$

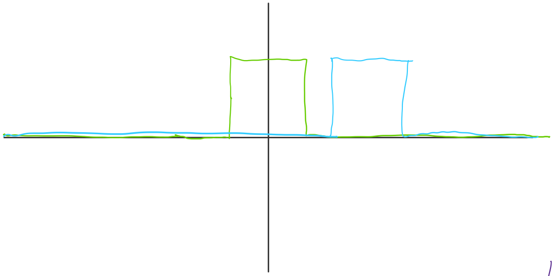
case 1 $t < -1$
case 4 $t > 1$ $\Big\} \rightarrow$ no overlap

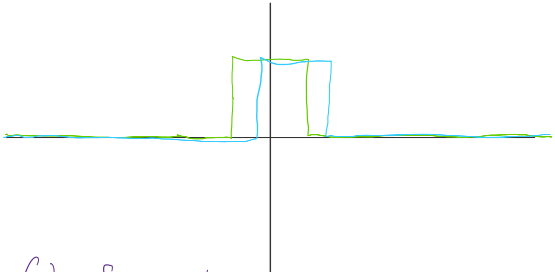
$-1 \le t \le 0 \rightarrow$ overlap in range $[-\frac{1}{2}, t+\frac{1}{2}]$, length $1+t = 1-|t|$

$0 \le t \le 1 \rightarrow$ overlap in range $[t-\frac{1}{2}, \frac{1}{2}]$ with length $1-t = 1-|t|$









$$\text{tri}(t) = \begin{cases} 1-|t| & |t| \le 1 \\ 0 & |t| > 1 \end{cases}$$

$$\text{rect}(t) * \text{rect}(t) = \int_{-\infty}^{\infty} \text{rect}(\tau)\, \text{rect}(t-\tau)\, d\tau$$ = length of interval where functions are both 1 $= \begin{cases} 0 & |t|>1 \\ 1-|t| & |t|<1 \end{cases} = \text{tri}(t)$

$|t-\tau| < \frac{1}{2}$

[redacted]