Exercises Chapter 3

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1.1)

$$(\sin m\omega t, \cos n\omega t) = \int_0^T \sin m\omega t \cos n\omega t \, dt = \int_{-\frac{T}{2}}^{\frac{T}{2}} \sin(m\omega t)\cos(n\omega t)\,dt = 0$$

, as $\sin(m\omega t)\cos(n\omega t)$ is an odd function (product of odd and even) integrated over interval symmetric around 0

1.2)

$$(\cos(m\omega t), \cos(n\omega t)) = \int_0^T \cos m\omega t \cos n\omega t \, dt = \int_0^T \tfrac{1}{2}\cos(m\omega t + n\omega t) + \tfrac{1}{2}\cos(m\omega t - n\omega t)\,dt = \frac{1}{2}\int_0^T \cos(\omega t(m+n)) + \cos(\omega t(m-n))\,dt = 0$$

(integrating over cosine for entire number of periods)

1.3)

$$(\sin(m\omega t), \sin(n\omega t)) = \int_0^T \sin(m\omega t)\sin(n\omega t)\,dt = \int_0^T \tfrac{1}{2}\left(\cos((m-n)\omega t) - \cos((m+n)\omega t)\right)dt = 0$$

(integrating over cosine for entire number of periods)

2)

$$f^{(n)}(t) = A_0 + \sum_{k=1}^{n}\left(A_k \cos k\omega_0 t + B_k \sin k\omega_0 t\right) = A_0 + \sum_{k=1}^{n}\left(\frac{A_k}{2}\left(e^{ik\omega_0 t} + e^{-ik\omega_0 t}\right) + \frac{B_k}{2i}\left(e^{ik\omega_0 t} - e^{-ik\omega_0 t}\right)\right) = A_0 + \sum\left(\left(\frac{A_k}{2} + \frac{B_k}{2i}\right)e^{ik\omega_0 t} + \left(\frac{A_k}{2} - \frac{B_k}{2i}\right)e^{-ik\omega_0 t}\right)$$

$$f^{(n)}t = \sum_{k=-n}^{n} C_k e^{ik\omega_0 t} = C_0 + \sum_{k=1}^{n}\left(C_k e^{ik\omega_0 t} + C_{-k} e^{-ik\omega_0 t}\right)$$

~~Obviously, k=0 gives (in eq. 3.28) the term $C_0 \cdot 1 = C_0$, so $A_0 = C_0$~~

Then $A_0 = C_0$

$$C_k = \frac{A_k}{2} + \frac{B_k}{2i} \rightarrow 2C_k = A_k - iB_k$$

$$C_{-k} = \frac{A_k}{2} - \frac{B_k}{2i} \rightarrow 2C_{-k} = A_k + iB_k$$