Exercises

Thursday, 14 November 2019 10:50

$$f(t) * \delta(t-t_0) = \int_{-\infty}^{\infty} f(\tau)\,\delta((t-t_0)-\tau)\,d\tau$$

$u = t - \tau + t_0$, $u = t - t_0 - \tau$, $\tau = t - t_0 - u$, $du = -d\tau$

$$= -\int_{\infty}^{-\infty} f(t-t_0-u)\,\delta(u)\,du$$

$$= \int_{-\infty}^{\infty} f(t-t_0-u)\,\delta(u)\,du$$

$$= f(t-t_0)$$

$$\int_{-\infty}^{\infty} f(t)\,\delta(t)\,dt = f(0)$$

$$\int_{-\infty}^{\infty} f(t-t_0)\,\delta(t)\,dt = f(t-t_0)$$

$$\int_{-\infty}^{\infty} f(u+t-t_0)\,\delta(u)\,du = f(t-t_0)$$

21.1) $\delta(t) * \delta(t) = \int_{-\infty}^{\infty} \delta(\tau)\,\delta(t-\tau)\,d\tau = \delta(t)$

21.2) $\delta(t-t_0) * \delta(t) = \int_{-\infty}^{\infty} \delta(\tau)\,\delta(t-t_0-\tau)\,d\tau = \delta(t-t_0)$

21.3) $\delta(t-t_0) * \delta(t-t_1) = \int_{-\infty}^{\infty} \delta(\tau-t_0)\,\delta(t-t_1-\tau)\,d\tau = -\int_{\infty}^{-\infty} \delta(u)\,\delta(t-t_1-u-t_0)\,du = \delta(t-t_1-t_0)$

21.4) $\frac{1}{t} * \delta(t-t_1) = \int_{-\infty}^{\infty} \frac{1}{\tau}\,\delta(t-t_1-\tau)\,d\tau = -\int_{\infty}^{-\infty} \frac{1}{-u+t-t_1}\,\delta(u)\,du = \int_{-\infty}^{\infty} \frac{1}{-u+t-t_1}\,\delta(u)\,du = \frac{1}{t-t_1}$

22) $f(t) * g(t) = \int_{-\infty}^{\infty} f(\tau)\,\frac{1}{a}\,\mathrm{rect}\left(\frac{t-\tau}{a}\right)d\tau = \frac{1}{a}\int_{t-\frac{1}{2}a}^{t+\frac{1}{2}a} f(\tau)\,d\tau$

which is the definition of average

$$\frac{t-\tau}{a} = \frac{1}{2}$$

$$t-\tau = \frac{1}{2}a$$

$$\tau = t - \frac{1}{2}a$$

23) ~~$f(t) * g(t) = \int_{-\infty}^{\infty} \ldots\, c\, d\tau =$~~

~~$\int_{-\infty}^{\infty} \ldots\, d\tau$~~

~~$\int_{-\infty}^{\infty} \ldots\, d\tau =$~~

$$f(t) * g(t) = c \cdot \int_{-\infty}^{\infty} g(\tau)\,d\tau = c = f_{average} \text{ if } \int_{-\infty}^{\infty} g(\tau)\,d\tau = 1$$

$= 1$ if $g(\tau)$ is normalized

24.1) $\cos(\omega t) * (1/T_0)\,\mathrm{rect}(t/T_0) = \frac{\sin(\omega T_0/2)}{\omega T_0/2}\cos(\omega t) = \mathrm{sinc}(\omega T_0/2)\cos(\omega t)$

24.2) $\omega = k2\pi/T_0 \rightarrow \mathrm{sinc}(\omega T_0/2)\cos(\omega t) = \mathrm{sinc}(k\pi)\cos\left(\frac{2k\pi t}{T_0}\right) = \frac{\sin(k\pi)}{k\pi}\cos\left(2k\pi\frac{t}{T_0}\right) = 0 \quad \forall k \in \mathbb{Z} \wedge k \neq 0$

24.3) $= 0 \rightarrow \mathrm{sinc}(\ldots) = 1$

$$\mathrm{sinc}(\omega T_0/2)\cos(\omega t) = \cos(\omega t)$$

$$\cos(\omega t) * \delta(t) = \cos(\omega t)$$

eq (2.43)

25) $f(t) * u(t) = \int_{-\infty}^{\infty} f(\tau)\,u(t-\tau)\,d\tau = \int_{-\infty}^{t} f(\tau)\,d\tau$

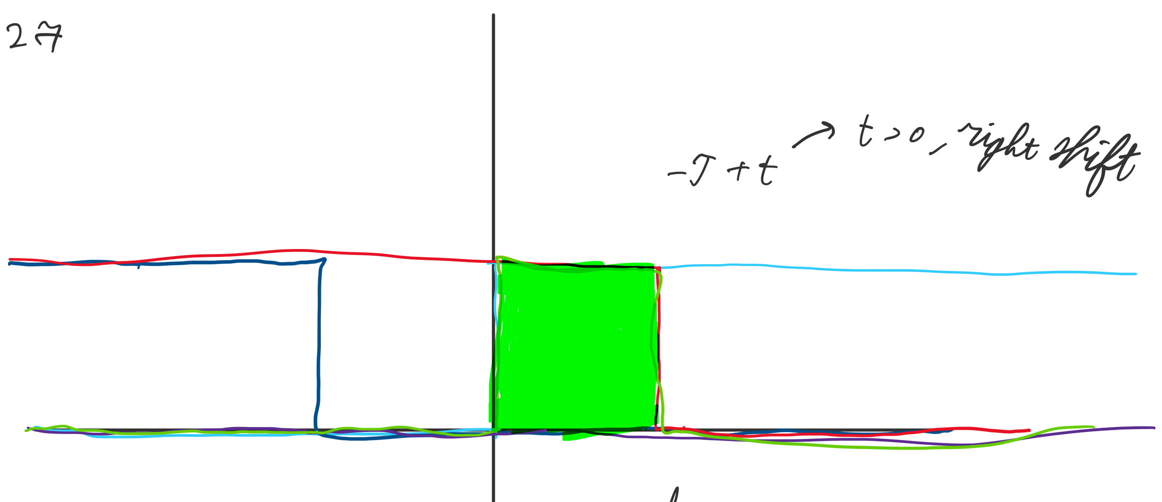
$t - \tau > 0 \rightarrow \tau < t$; for $t > \tau$, $u(t-\tau) = 1$, else 0

26) to prove: $f(t) = \sum_{k=-\infty}^{\infty} g(t-kT_0)$

$$g(t) * \delta_{T_0}(t) = \int_{-\infty}^{\infty} g(\tau)\left(\sum_{k=-\infty}^{\infty}\delta(t-kT_0-\tau)\right)d\tau = -\int_{\infty}^{-\infty}\sum_{k=-\infty}^{\infty} g(t-kT_0-u)\,\delta(u)\,du =$$

$$\sum_{k=-\infty}^{\infty} g(t-kT_0) = f(t)$$

27



for $t > 0$, $A = t$

for $t < 0$, $A = 0$