Lecture

Thursday, 14 November 2019 08:47

$$\delta(t-t_0)=0 \quad \text{for } t\neq t_0$$

$$\int_{-\infty}^{\infty} f(t)\delta(t-t_0)\,dt=f(t_0)$$

$$\int_{-\infty}^{\infty} \delta(t-t_0)\,dt=1$$

$$f(t)\,\delta(t-t_0)=f(t_0)\,\delta(t-t_0)$$

convolution product

$$f(t) * g(t)=\int_{-\infty}^{\infty} f(\tau)\,g(t-\tau)\,d\tau$$

$$[f*g](t)$$

commutative $f*g=g*f$

associative $(f*g)*h=f*(g*h)$

distributive $f*(g+h)=f*g+f*h$

$$f(t) * \delta(t-t_0)=\int_{-\infty}^{\infty} f(\tau)\,\delta(t-t_0-\tau)\,d\tau=f(t-t_0)$$

$$f(t) * \delta(t)=f(t)$$

$\delta(t)$ is the 'unity element' of the convolution product

$$\delta(t) * \delta(t)=\delta(t)$$

$$\delta(t) * \delta(t-t_0)=\delta(t-t_0)$$

$$\delta(t-t_0) * \delta(t-t_1)=\delta(t-t_1-t_0)$$

$$\frac{1}{t} * \delta(t-t_1)=\frac{1}{t-t_1}$$

$$f(t) * D(\varepsilon,t)=\int_{-\infty}^{\infty} f(\tau)\,D(\varepsilon,t-\tau)\,d\tau=\frac{1}{\varepsilon}\int_{t-\frac{\varepsilon}{2}}^{t+\frac{\varepsilon}{2}} f(\tau)\,d\tau$$

should be sign-pointed

This is the average value of f

When $g(t)$ is normalized, then $f(t)*g(t)$ is a weighted average of $f(t)$

popular choice $g(t)=\frac{1}{a\sqrt{2\pi}}e^{-t^2/2a^2}$

cross-correlation

$$f(t) \star g(t)=\int_{-\infty}^{\infty} f(t)\,g(t+\tau)\,d\tau$$

auto correlation

$$f(t) \star g(t)=\int_{-\infty}^{\infty} f(t)\,f(t+\tau)\,d\tau$$

$f(t) \star f(t)$ will peak at $t=kT$ ← period

in general: with $f(t) \star f(t)$ we can detect periodicity

Cross-correlation will peak for shifts that make f and g as similar as possible

- → template matching
- → facial recognition

$$\underline{V}=V_x\underline{e_x}+V_y\underline{e_y}+V_z\underline{e_z}$$

$$V\cdot e_x=V_x\underline{e_x}\cdot\underline{e_x}+0$$

complex conjugate

$$\|f-g\|^2=\int_0^{T_0}|f-g|^2\,dt=\int_0^{T_0}(f-g)(f-g)^*\,dt$$

$$(f,g)=\int_0^{T_0} f(t)\,g^*(t)\,dt$$

$$\|f\|^2=(f,f)$$

$(f,g)=0 \rightarrow$ f and g orthogonal

Let $b_k(t)$ be orthogonal functions on the interval

$$f^{(n)}(t)=\sum_{k=0}^{n} f_k\,b_k(t)$$

$f_k=\frac{(f,b_k)}{\|b_k\|^2}$ is the best approximation

$$f(t)=\sum_{k=-\infty}^{\infty} C_k e^{ik\omega_0 t}$$

$$C_k=\frac{(f,e^{ik\omega_0 t})}{\|e^{ik\omega_0 t}\|^2}=\frac{1}{T_0}\int_{T_0} f(t)e^{-ik\omega_0 t}\,dt$$

An aperiodic function is a periodic function with $T_0\rightarrow\infty$

$$f(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\,e^{i\omega t}\,d\omega$$

$$F(\omega)=\int_{-\infty}^{\infty} f(t)e^{-i\omega t}\,dt$$

Fourier transform $F(\omega)$ is the spectrum of $f(t)$

$$F(\omega)=\delta(\omega-\omega_0)$$

$$f(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\,e^{i\omega t}\,d\omega=\frac{1}{2\pi}\ ?$$