Exercises Chapter 5

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1) $$F_s(\omega) = \mathcal{F}\{f_s(t)\} = \mathcal{F}\{f(t)\delta_{T_s}(t)\}$$

$$= \frac{1}{2\pi}\mathcal{F}\{f(t)\} * \mathcal{F}\{\delta_{T_s}(t)\}$$

$$= \frac{1}{2\pi}\cancel{\int_{-\infty}^{\infty}} F(\omega) * \frac{2\pi}{T_s}\delta_{\frac{2\pi}{T_s}}(\omega)\, d\omega$$

? this is not the convolution product, is it?

$$= \frac{1}{T_s}\cancel{\int_{-\infty}^{\infty}} F(\omega) * \delta_{\frac{2\pi}{T_s}}(\omega)\, d\omega$$

$$= \frac{1}{T_s}\cancel{\int_{-\infty}^{\infty}} F(\omega) * \sum_{k=-\infty}^{\infty}\left(\omega - k\frac{2\pi}{T_s}\right) d\omega$$

$$= \frac{1}{T_s}\cancel{\int_{-\infty}^{\infty}} F(\omega) * \sum_{k=-\infty}^{\infty}(\omega - k\omega_s)\, d\omega$$

$$= \frac{1}{T_s}\sum_{k=-\infty}^{\infty} F(\omega - k\omega_s)$$

2) $$f(t) = f_s(t) * \operatorname{sinc}(\omega_s t/2)$$

$$= \left(f(t)\delta_{T_s}(t)\right) * \operatorname{sinc}(\omega_s t/2)$$

$$= \left(f(t)\sum_{k=-\infty}^{\infty}\delta(t - kT_s)\right) * \operatorname{sinc}(\omega_s t/2)$$

$$= \int_{-\infty}^{\infty} f(\tau)\sum_{k=-\infty}^{\infty}\delta(\tau - kT_s)\operatorname{sinc}(\omega_s(t-\tau)/2)$$

replace by

$$= \sum_{k=-\infty}^{\infty} f(kT_s)\operatorname{sinc}(\omega_s(t - kT_s)/2)$$

replacing values only leaves contributions for integer k

3) $$F_s(\omega) = \mathcal{F}\{f_s(t)\} = \mathcal{F}\{f(t)\delta_{T_s}(t)\}$$

$$= \int_{-\infty}^{\infty} f(t)\delta_{T_s}(t)\, e^{-i\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} f(t)\sum_{k=-\infty}^{\infty}\left(\delta(t - kT_s)\right) e^{-i\omega t}\, dt$$

$$= \sum_{n=-\infty}^{\infty} f(nT_s)\, e^{-i\omega n T_s}$$