Lecture

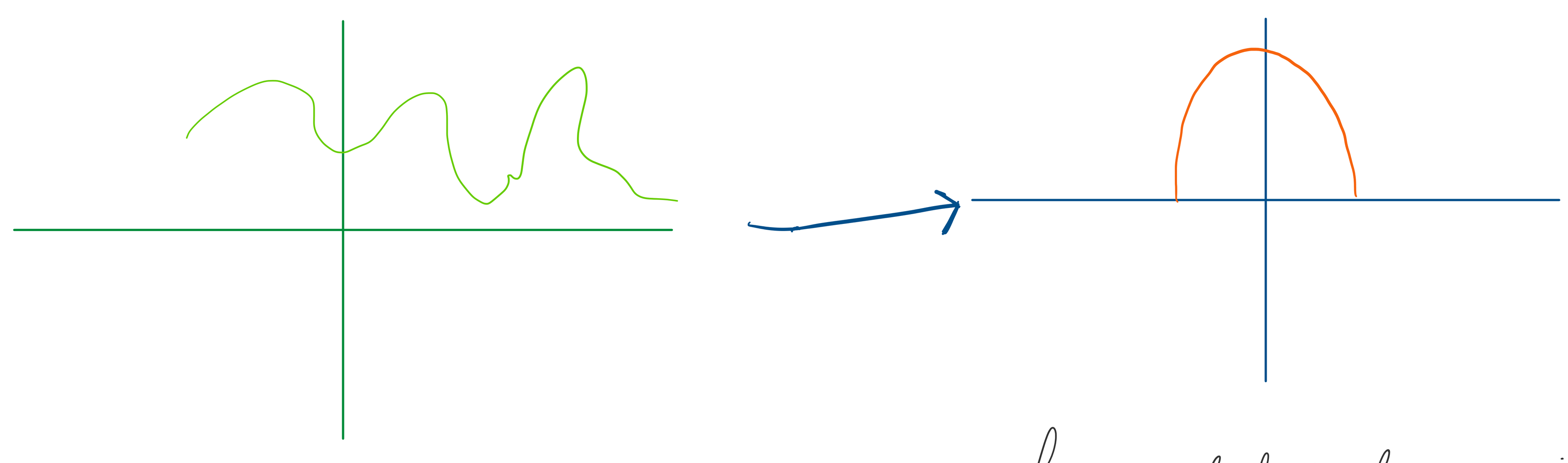
Tuesday, 19 November 2019 13:30

$$f(t) * \delta(t - t_0) = f(t - t_0)$$

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega) e^{i\omega t}\, d\omega$$

$$e^{i\omega t} = \cos(\omega t) + i \sin(\omega t)$$

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t}\, dt$$



how much of a given frequency is in the signal

$$f(t) * g(t) \overset{\mathcal{F}}{\longleftrightarrow} F(\omega) G(\omega)$$

$$f(t) g(t) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{2\pi} F(\omega) * G(\omega)$$

$f[k] \xrightarrow{DFT} F[m]$

$f[k] \leftrightarrow f(kT_s)$, $F[m] \leftrightarrow F_s(\omega)$

impulse train: $\delta_T(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT)$

ideal sampling of a signal $f_s(t) = f(t)\, \delta_{T_s}(t)$

$$\int_{-\infty}^{\infty} f_s(t)\, dt = \sum_{k=-\infty}^{\infty} \int_{-\infty}^{\infty} f(t)\, \delta(t - kT_s)\, dt = \sum_{k=-\infty}^{\infty} f(kT_s)$$

$$\mathcal{F}\{\delta_{T_0}(t)\} = \frac{2\pi}{T_0} \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_0) = \omega_0 \delta_{\omega_0}(\omega)$$

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spectrum $F_s(\omega)$ of $f_s(t) = f(t)\, \delta_{T_s}(t)$

$$F_s(\ ) = \mathcal{F}\{f(t)\, \delta_{T_s}(t)\} = \frac{1}{2\pi} F(\omega) * \omega_s \delta_{\omega_s}(\omega)$$

$$F_s(\omega) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} F(\omega - k\omega_s)$$

Every $f_s(t)$ has a periodic spectrum

bandlimited signal $\omega \le \omega_b$: no overlap if $\omega_s > 2\omega_b$

$$F(\omega) = 2\pi\, \omega_s^{-1} F_s(\omega)\, \mathrm{rect}(\omega/\omega_s)$$

$\omega_s \le 2\omega_b \rightarrow$ aliasing $\rightarrow$ $F(\omega)$ can no longer be reconstructed!

$\omega_s = 2\omega_b$ → Nyquist frequency

$F(\omega)$ can be reconstructed from $F_s(\omega)$ if $\omega_s > 2\omega_b$

else; aliasing

if $\omega_s > 2\omega_b$

$$f(t) = \sum_{k=-\infty}^{\infty} f[k]\, \mathrm{sinc}\left(\omega_s (t - kT_s)/2\right)$$

resampling : $g[k] = f(kT_k)$

$$F_s(\omega) = \sum_{m=-\infty}^{\infty} \int_{-\infty}^{\infty} f(t)\, \delta(t - mT_s)\, e^{-i\omega t}\, dt = \sum_{m=-\infty}^{\infty} f[m] e^{-i\omega m T_s}$$

normalized frequency: $\Omega = \omega T_s$

$$F(\Omega) = F_s\left(\frac{\Omega}{T_s}\right) = \sum_{m=-\infty}^{\infty} f[m] e^{-im\Omega}$$

DTFT

Let $f[m] = 0$ for $m < 0$, $m > N$

$$F(\Omega) = \sum_{m=0}^{N-1} f[m] e^{-im\Omega}$$

$$F[k] = F_N(2\pi k/N) = \sum_{m=0}^{N-1} f[m] e^{-i 2\pi k m/N} \qquad k = 0, 1, 2, \ldots, N-1$$

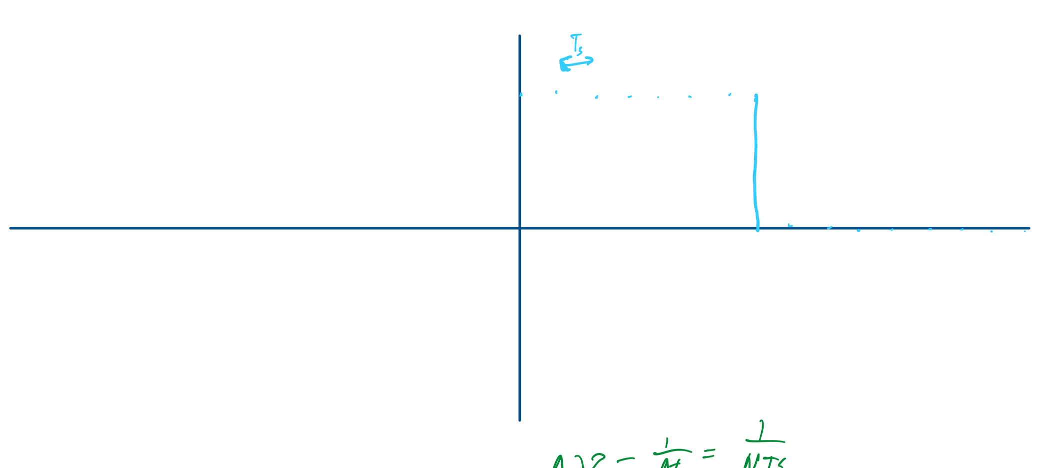
DFT

→ approximates $F_s(\omega)$ at $\omega_k = \omega_s k/N$

no aliasing present: $F(\omega_k) = T_s F_s(\omega)$

$$\Delta\omega = \omega_s / N = 2\pi / N T_s$$

$$\Delta\nu = \frac{1}{\Delta t}$$



$$\Delta\nu = \frac{1}{\Delta t} = \frac{1}{NT_s}$$

$$\Delta\omega = 2\pi\, \Delta\nu = \frac{2\pi}{NT_s} = \frac{2\pi}{\Delta T}$$

$$\omega_{max} = \frac{\omega_s}{2} = \frac{\pi}{T_s}$$

$$\omega = \frac{2\pi}{T}$$

no aliasing: $\omega_{max} \gg \frac{2\pi}{T} \Rightarrow T \gg T_s \Rightarrow L \gg 1$

good resolution: $\Delta\omega \ll \frac{2\pi}{T} \Rightarrow T \ll \Delta t \Rightarrow L \ll N$

good results require that $1 \ll L \ll N$