Lecture
Monday, 23 November 2020 10:41

$f \xrightarrow{\mathcal{F}} F[\cdot]$

$f(t) \qquad F_1\left(\omega, \frac{2\pi}{N}\right)$

if we always (in reality, filtered, belongings), the $F[\cdot]$ contains the spectrum $F(\omega)$ of $f(t)$

write impedance $Z(\omega)$
transfer function $H(\omega)$

signal $m(t) = A(t)\cos(\omega_c t)$
multiply with $c(t) = \cos(\omega_c t)$

$$x(t) = m(t)\,c(t) = A(t)\,\frac{1}{2}\left(\cos(\omega_c + \omega_c)t + \cos(\omega_c - \omega_c)t\right)$$

multiplication of harmonic signal $\longrightarrow$ sum of signals with sum and difference frequencies

now, take a arbitrary signal $m(t)$

$$x(\omega) = m(t)\,c(t) \xrightarrow{\mathcal{F}} X(\omega) = \frac{1}{2\pi} M(\omega) * C(\omega)$$

$$C(t) = \mathcal{F}\{\cos(\omega_c t)\} = \pi\left(\delta(\omega - \omega_c) + \delta(\omega + \omega_c)\right)$$

$$X(\omega) = \frac{1}{2}\left(M(\omega - \omega_c) + M(\omega + \omega_c)\right)$$

radio : transmitted $x(t) = x_1(t) + x_2(t)$ with $X(\omega) = X_1(\omega) + X_2(\omega)$

use low-pass filter like $H(\omega) = \mathrm{rect}(\omega/2\omega_c)$

$$S(\omega) = \frac{A_1 A_2}{2}\left(M(\omega - \omega_c) + M(\omega + \omega_c)\right) + A_1 \pi\left(\delta(\omega - \omega_c) + \delta(\omega + \omega_c)\right)$$

$$H(\omega) = \mathrm{rect}(\omega / 2\omega_c)$$

ideal low-pass filter $\omega > \omega_c$ is eliminated

resistor $v_i(t) - v_r(t) = R\,I(t)$

coil $v_i(t) - v_r(t) = L\,\frac{dI}{dt}$

capacitor $v_i(t) - v_o(t) = \frac{1}{C}\int_{-\infty}^{t} I(t)\,dt$

$$\mathcal{F}\left(\frac{df(t)}{dt}\right) = i\omega F(\omega)$$

$$\mathcal{F}\left\{\int_{-\infty}^{t} f(t)\,dt\right\} = \left(\frac{1}{i\omega} + \pi\delta(\omega)\right) F(\omega) = \frac{F(\omega)}{i\omega} + \pi F(0)\delta(\omega)$$

$\boxed{F(0) \text{ is the DC component of } f(t)}$

$$V_i(\omega) - V_r(\omega) = R\,I(\omega)$$
$$V_i(\omega) - V_r(\omega) = i\omega L\,I(\omega)$$
$$V_i(\omega) - V_o(\omega) = \frac{I(\omega)}{i\omega C} + \frac{\pi}{C} I(0)\delta(\omega)$$

$$V_i(\omega) - V_o(\omega) = Z(\omega)\,I(\omega)$$

Z is the impedance of the element

no DC current: for C we have $V_i(\omega) - V_o(\omega) = \frac{I(\omega)}{i\omega C}$

R	$Z = R$
L	$Z = i\omega L$
C	$Z = \frac{1}{i\omega C}$

in series: add impedances $Z_{tot} = \sum_{k=1}^{n} Z_k$
in parallel: $\frac{1}{Z_{tot}} = \sum_{k=1}^{n} \frac{1}{Z_k}$

$Z_s = Z_1 + Z_2$
$\frac{1}{Z_p} = \frac{1}{Z_1} + \frac{1}{Z_2}$

$$V_1 = I\,(Z_1 + Z_2)$$
$$V_2 = I\,Z_2$$

$$H = \frac{V_2}{V_1} = \frac{Z_2}{Z_1 + Z_2}$$

let $Z_1 = i\omega L + \frac{1}{i\omega C}$
$Z_2 = R$

$$\tau_{RC} = RC$$
$$\tau_{LC} = \sqrt{LC}$$

$$H(\omega) = \frac{Z_2}{Z_1 + Z_2} = \frac{i\omega RC}{1 - \omega^2 LC + i\omega RC}$$

$$|H(\omega)| = \frac{\omega RC}{\sqrt{(1 - \omega^2 LC)^2 + \omega^2 (RC)^2}}$$

Bode diagram $|H(\omega)|$ logarithmic
$\phi(\omega)$ linear

no current passes through the LC-combo for low or high ω