Exercises Chapter 11

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$T[\alpha x_1 + \beta x_2] = \alpha T[x_1] + \beta T[x_2]$ linearity

$y(t) = T[x(t)] \Leftrightarrow y(t-\tau) = T[x(t-\tau)]$ time invariance

impulse response $h(t) \equiv T[\delta(t)]$ output of input delta function

$h(t-\tau) \equiv T[\delta(t-\tau)]$

impulse response of integrator is unit step function (Heaviside)

$\boxed{11.1}$

$$K[\delta(t-t_0)] = k(t)\delta(t-t_0)$$
$$= k(t-t_0)\delta(t-t_0) \rightarrow \text{because } \delta(t-t_0) = 0 \text{ if } t \neq t_0 \text{ and } k \text{ is constant}$$
$$= h(t-t_0)$$

$$I[\delta(t-t_0)] = \int_{-\infty}^{t} \delta(\tau - t_0)d\tau = \begin{cases} 0 & \text{if } t < t_0 \\ 1 & \text{if } t > t_0 \end{cases} = u(t) = h(t)$$

This is to be expected because the systems are time-invariant

$\boxed{11.2}$

$$h(t) = K(t)\delta(t) = k(0)\delta(t) \text{ because } \delta(t) = 0 \text{ if } t \neq 0$$

$$K[\delta(t-t_0)] = y_{t_0}(t) = K(t-t_0)\delta(t-t_0) = K(t_0)\delta(t-t_0) \text{ because } \delta(t-t_0) = 0 \text{ if } t \neq t_0$$

$$y_{t_0}(t) = K(t_0)\delta(t-t_0) \neq K(0)\delta(t-t_0) = h(t-t_0)$$

For LTI system $y(t) = h(t) * x(t)$

$\boxed{11.3}$

$$y(t) = \frac{1}{2a}\int_{t-a}^{t+a} x(\tau)d\tau$$

$$h(t) = \frac{1}{2a}\int_{t-a}^{t+a} \delta(\tau)d\tau$$

$$= \frac{1}{2a}\begin{cases} 1 & \text{if } \cancel{t>0}\ |t| \leq a \\ 0 & \text{if } \cancel{t<0}\ |t| > a \end{cases} = \frac{1}{2a}\,\mathrm{rect}\left(\frac{t}{2a}\right)$$

$\hookrightarrow h(t) \neq 0$ for $t \in (-a, 0]$, hence not causal / not realisable

$$= \frac{1}{2a}u(t)$$
$$= \frac{u(t)}{2a}$$

~~This system is causal as $h(t) = 0$ for $t < 0$. Hence, it can be realised in practice~~