Lecture
Thursday, 28 November 2019 08:49

Euclid's five postulates

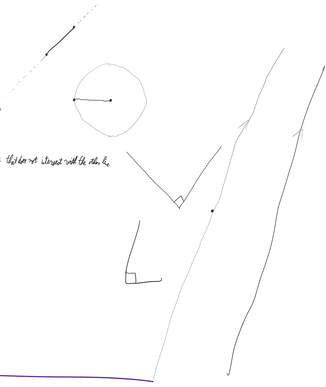
1 a straight line segment can be drawn joining any pair of two points

2 a straight line segment can be extended indefinitely into a long line

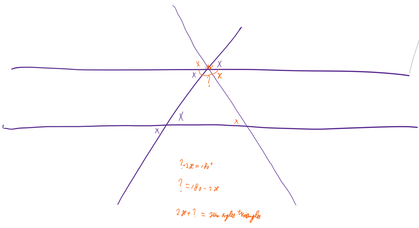
3 a line segment is the radius of a circle with one of the ends being the center

4 all right angles are equal → all straight angles are equal

5 given a line and a point not on the line, there is only one line through that point that does not intersect with the other line



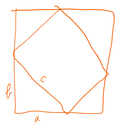
Corollary:
- Z-corners
- sum of angles of a triangle is 180°
- Pythagoras



$? = x = 45°$

$? = 180 - 3x$

$2x + ? = 90$ right triangle



$(a+b)^2 = \text{total}$

$\frac{1}{2} a b \cdot 4 + c^2 = \text{total}$

$2ab + c^2 = (a+b)^2 = a^2 + b^2 + 2ab$

$a^2 + b^2 = c^2$

$a = b$ □

geometrical objects: no coordinate system

$\underline{v} = \frac{d\underline{x}}{dt}$ → no coordinate system

$\underline{a} + \underline{b} = \underline{b} + \underline{a}$ → vector addition is commutative only in Euclidean space

$a \cdot b = \|a\| \, \|b\| \cos \measuredangle_{ab}$

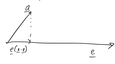
$a \cdot a = \|a\|^2 \Longrightarrow \|a\| = \sqrt{a \cdot a}$

properties: bilinear
commutative
distributive

$\underline{c} = \underline{a} + \underline{b}$

$(\underline{a} + \underline{b})(\underline{a} + \underline{b}) = \|\underline{a}\|^2 + \|\underline{b}\|^2 + 2\,\underline{a}\,\underline{b}$

$e \, (a \cdot e)$ → projection of a onto e



surface of parallelogram $a, \underline{b}$

$\|a \times b\| = \|a\| \, \|b\| \sin \measuredangle_{ab}$

$\underline{a} \times \underline{a} = \underline{0}$

bilinear
anti-commutative
distributive

$\underline{a} \times \underline{b} = -\underline{b} \times \underline{a}$

Scalar triple product

$(a \;\; b \;\; c) = a \cdot (b \times c)$

trilinear
anti-commutative
distributive

Scalar triple product → volume

$\underline{u} = \sum_{i=1}^{3} u_i \, \underline{a}_i$

$\underline{u} = \sum_{i=1}^{3} u^i \underline{a}_i = u^i \underline{a}_i$

$\underline{u} = u^i a_i$ → notation implies summation

$\underline{u} = u^i \underline{a}_i \qquad v = v^j a_j$ — we allow indices for different objects

$\underline{u} \cdot \underline{v} = u^i v^j \, \underline{a}_i \cdot a_j$

$g_{ij} = \underline{a}_i \cdot a_j$ — the components of the metric tensor, an $n \times n$-matrix

$\underline{u} \cdot \underline{v} = u^i v^j g_{ij} \Rightarrow \|u\| = \sqrt{u^i u^j g_{ij}}$

"translates coordinate interval into distance"