Lecture

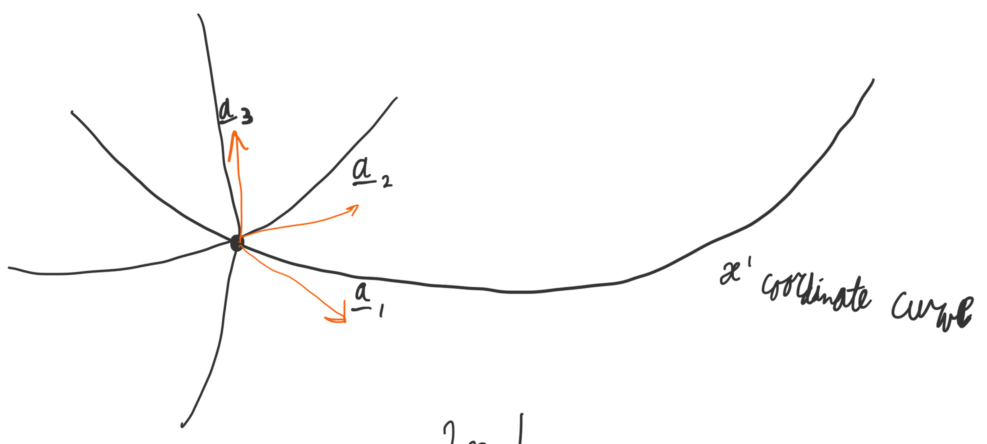
Tuesday, 10 December 2019 13:31

curvilinear coordinate systems

$\underline{x}\,(x^1, x^2, x^3)$

$\underline{x}\,(x_0^1, x_0^2, x_0^3)$

$x\,(x^1, x_0^2, x_0^3) \rightarrow$ curve



$\left.\frac{\partial x}{\partial x^1}\right|_0 = a_1$

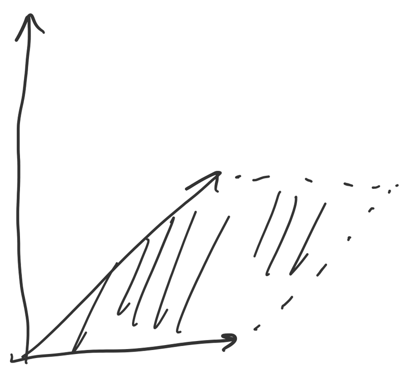
$\frac{\partial x}{\partial x^1} = a_1$

$V = (\underline{a}_1\ \underline{a}_2\ \underline{a}_3) = (\underline{a}_1 \times \underline{a}_2)\cdot \underline{a}_3 = \sqrt{g} = \sqrt{\det g} = \sqrt{\det(\text{metric tensor})}$

$$g = \begin{pmatrix} a_1\cdot a_1 & a_1\cdot a_2 & a_1\cdot a_3 \\ a_2\cdot a_1 & a_2\cdot a_2 & a_2\cdot a_3 \\ a_3\cdot a_1 & a_3\cdot a_2 & a_3\cdot a_3 \end{pmatrix}$$

$dV = (a_1\, dx^1 \times a_2\, dx^2)\cdot a_3\, dx^3$
$= (a_1 \times a_2)\cdot a_3\, dx^1 dx^2 dx^3$
$= \sqrt{g}\, dx^1 dx^2 dx^3$

$\int_V f(x)\, dV = \iiint_{x^3 x^2 x^1} f(x(x^1, x^2, x^3))\, \sqrt{g(x^1, x^2, x^3)}$ . . . .



$A^3 = \underline{a}_1 \times \underline{a}_2$

in general $A^3 \neq a^3$

$\underline{A}^3 = \sqrt{g}\ \underline{a}^3$

$\underline{a}^3 = \frac{1}{\sqrt{g}} \underline{A}^3 = \frac{1}{\sqrt{g}} \underline{a}_1 \times \underline{a}_2$

Contravariant basis:

$\underline{a}^1 = \frac{1}{\sqrt{g}} \underline{a}_2 \times \underline{a}_3$

$\underline{a}^2 = \frac{1}{\sqrt{g}} \underline{a}_3 \times \underline{a}_1$

$a^i \cdot a_j = \delta^i_j = \begin{cases} 1 & \text{if } i=j \\ 0 & \text{if } i\neq j \end{cases}$

$\underline{u} = \underbrace{u^i \underline{a}_i}_{} = \underbrace{u_i \underline{a}^i}_{}$

summations are implied (Einstein)

$a^i \cdot a^j \equiv g^{ij}$

$g^{ij} g_{jk} = \delta^i_k$

$u_i = g_{ij}\, u^j$
$\mathbf{a_i = g_{ij}\, a^j}$
$u_i = g^{ij} a_j$
$a^i = g^{ij} a_j$

$\underline{u} = u^i \underline{a}_i = u_i \underline{a}^i$
$u^i \underline{a}_i \cdot \underline{a}_j = u_i \underline{a}^i \cdot \underline{a}_j = u_j$
$u_j = u^i g_{ij}$

$u^i a_i = u_i a^i = u^{(i)} e_{(i)} = u_{(i)} e^{(i)}$

OCL: $a_i \,//\, a^i \,//\, e_i$