Lecture

Thursday, 12 December 2019 08:46

$\underline{u} = u^i \underline{e}_i = u_i \underline{e}^i = u_{(i)} \underline{e}^{(i)} = u^{(i)} \underline{e}_{(i)}$

u^i

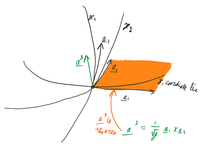
u_i

$u_{(i)}$

$u^{(i)}$

$\underline{x}(x^1, x^2, x^3)$

$\underline{e}_i = \frac{\partial \underline{x}}{\partial x^i}$



$\partial \underline{x} = \frac{\partial \underline{x}}{\partial x^i} \partial x^i = \underline{e}_i \partial x^i$

$\boxed{\text{gradient \& divergence}}$

example:

$d\underline{A} = \underline{n} dA$

$\rho(\underline{x}, t)$ $\qquad$ $\rho(x, t) dA$

$\underline{v} = \frac{d\underline{x}}{dt}$

$d\phi = \rho \, d\underline{A} \cdot d\underline{x} = \rho \underline{v} \cdot d\underline{A} \, dt$

$= \underline{\Gamma} \cdot d\underline{A} \, dt = \underline{\Gamma} \cdot \underline{n} \, dA \, dt$

$\frac{d\phi}{dA \, dt} = \underline{\Gamma} \cdot \underline{n} = \Gamma_n$

flux density

$\phi_A = \int_A \underline{\Gamma} \cdot d\underline{A} = \int_A \underline{\Gamma} \cdot \underline{n} \, dA = \int_A \Gamma_n \, dA$

$\frac{d}{dt} \int_V \rho \, dV = - \int (\rho \underline{v} \cdot \underline{n}) dA$

$\frac{d}{dt} \int_V \rho(\underline{x}, t) dV + \oint_{\partial V} (\rho \underline{v} \cdot \underline{n}) dA = 0$

density

$\underline{\Gamma} = n_s \underline{v}$

volumetric production rate

species mass balance $\boxed{\frac{d}{dt} \int_V n_s dV = - \int_{\partial V} \underline{\Gamma}_s \cdot \underline{n} \, dA + \int_V s_s dV}$

$\frac{1}{V} \int_{\partial V} \underline{\Gamma}_s \cdot \underline{n} \, dA = \frac{1}{V} \int_V s_s \, dV = \langle s_s \rangle$

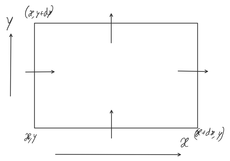
average of s_s

$\lim_{V \to 0} \frac{1}{V} \int_V s_s \, dV = \boxed{\lim_{V \to 0} \frac{1}{V} \int_{\partial V} \underline{\Gamma}_s \cdot \underline{n} \, dA} = \nabla \cdot \underline{\Gamma}_s$

divergence of flux vectors to density

scalar

divergence



$\frac{df}{dx} = \frac{f(x+\partial x) - f(x)}{\partial x}$

$\Gamma_x (x+dx, y) \, dy \, dz - \Gamma_x (x, y) dy \, dz \approx \frac{\partial \Gamma_x}{\partial x} dx \, dy \, dz = \frac{\partial \Gamma_x}{\partial x} dV$

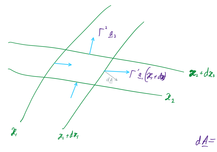
$\int \underline{\Gamma} \cdot \underline{n} \, dA = \left(\frac{\partial \Gamma_x}{\partial x} + \frac{\partial \Gamma_y}{\partial y} + \frac{\partial \Gamma_z}{\partial z} \right) dV$

$\frac{1}{dV} \int \underline{\Gamma} \cdot \underline{n} \, dA = \left(\frac{\partial \Gamma_x}{\partial x} + \frac{\partial \Gamma_y}{\partial y} + \frac{\partial \Gamma_z}{\partial z} \right)$

$\lim_{dV \to 0} \frac{1}{dV} \int \underline{\Gamma} \cdot \underline{n} \, dA = \left(\frac{\partial \Gamma_x}{\partial x} + \frac{\partial \Gamma_y}{\partial y} + \frac{\partial \Gamma_z}{\partial z} \right)$

$\nabla \cdot \underline{\Gamma} = \frac{\partial \Gamma_x}{\partial x} + \frac{\partial \Gamma_y}{\partial y} + \frac{\partial \Gamma_z}{\partial z}$

$\underline{x}(x^1, x^2, x^3)$



$d\underline{A} = \sqrt{g} \, \underline{e}^1 dx^2 dx^3$

$\underline{e}_i \cdot d\underline{A} = \partial V$

important

$\Gamma^1 \underline{e}_1 \cdot \sqrt{g} \, \underline{e}^1 dx^2 dx^3 = \sqrt{g} \, \Gamma^1 dx^2 dx^3$

$\sqrt{g} \left(\Gamma^1 (x_1 + \partial x_1, \ldots) - \Gamma^1 (x_1, \ldots) \right) dx^2 dx^3$

$\frac{1}{\sqrt{g}} \frac{\partial \sqrt{g} \Gamma^1}{\partial x^1} \sqrt{g} \, dx^1 dx^2 dx^3 = \frac{1}{\sqrt{g}} \frac{\partial \sqrt{g} \Gamma^1}{\partial x^1} dV$

derivative might appear here

$\nabla \Gamma = \frac{1}{\sqrt{g}} \frac{\partial (\sqrt{g} \Gamma^1)}{\partial x^1}$

r, θ, φ

$\frac{\partial}{\partial \varphi} \cancel{r} \theta \, f(\ldots) = r \theta \frac{\partial}{\partial \varphi} f(\ldots)$