Lecture

Tuesday, 17 December 2019 13:26

gradient

$\nabla \stackrel{\text{grad}}{=} \left(\partial/\partial x , \partial/\partial y , \partial/\partial z \right)$

$\underline{x}(x^1, x^2, x^3)$

covariant $\underline{a}_i = \frac{\partial \underline{x}}{\partial x^i}$

contravariant

$a_i \cdot a^j = \delta^i_j$

$T(\underline{x}, t)$

$df = \frac{\partial f}{\partial x^i} dx^i$

"superscript in denominator = subscript"

$d\underline{x} = \frac{\partial \underline{x}}{\partial x^j} dx^j$

$= \underline{a}_j dx^j$

$\underline{a}^i \cdot d\underline{x} = \underline{a}^i \cdot \underline{a}_j dx^j$

$= \delta^i_j dx^j$

$= dx^i$

$df = \frac{\partial f}{\partial x^i} \underline{a}^i \cdot d\underline{x} = \nabla f \cdot d\underline{x}$

$\nabla f = \frac{\partial f}{\partial x^i} \underline{a}^i$ **was in exam!**

(derivation)

$\|\underline{a}^i\| = \frac{1}{h_i}$ OCL

$\|\underline{a}_i\| = h_i$

$\nabla f = \frac{\partial f}{\partial x^i} \underline{a}^i \stackrel{OCL}{=} \|\underline{a}^i\| \frac{\partial f}{\partial x^i} \frac{\underline{a}^i}{\|\underline{a}^i\|} = \|\underline{a}^i\| \frac{\partial f}{\partial x^i} e^{(i)} = [\nabla f]_{(i)} \underline{e}^{(i)} = [\nabla f]^{(i)} \underline{e}_{(i)}$

$= \frac{1}{h_i} \frac{\partial f}{\partial x^i} e^{(i)}$

$\left.\begin{matrix} T(\underline{x}(t)) \\ x(t) \end{matrix}\right\} T(\underline{x}(t), t)$

$\frac{dT}{dt}(\underline{x}(t), t) = \frac{dT(\underline{x}(x^1(t), x^2(t), x^3(t)), t)}{dt}$

$= \frac{\partial T}{\partial x^i} \frac{dx^i}{dt} + \frac{\partial T}{\partial t}$

T - change in position

T - change in time

$\boxed{\text{different! may cost a lot of points!}}$

$= \nabla T \cdot \frac{d\underline{x}}{dt} + \frac{\partial T}{\partial t}$

$\boxed{= \frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T = \frac{DT}{Dt}}$ material derivative

$dT = \nabla T \cdot d\underline{x}$

$df = \nabla f \cdot d\underline{x} = \nabla f \cdot \underline{u}\, ds$

direction vector (length 1)

$d\underline{x} = \underline{u}\, ds$

$ds = \|d\underline{x}\|$

$\frac{df}{ds} = \nabla f \cdot \underline{u}$

$\boxed{\frac{df}{ds} = \nabla f \cdot \underline{u}}$ → directional derivative of f in the direction of $\underline{u}$

other ways of writing $\boxed{D_u(f) = \frac{\partial f}{\partial u} = \nabla f \cdot \underline{u}}$

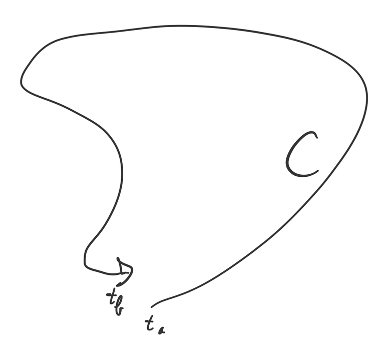
The normal derivative $\frac{\partial f}{\partial \underline{n}} = \nabla f \cdot \underline{n}$ normal vector (to surface)

heat flux through boundary

Line Integrals

$I = \int_C \underline{u}(\underline{x}) \cdot d\underline{x} = \int_{t_0}^{t_f} \underline{u}(\underline{x}(t)) \cdot \underline{x}'(t)\, dt$

velocity of curve



parameterize

$\underline{x}(t)$

$d\underline{x} = \frac{d\underline{x}}{dt} dt = x'(t)\,dt$

assume $\underline{u} = \nabla f$

$I = \int_C \underline{u}(\underline{x}) \cdot d\underline{x} = \int_C \nabla f \cdot d\underline{x} = \int_C df = f(\underline{x}_f) - f(\underline{x}_0)$

$\underline{F} \cdot d\underline{x} \rightarrow$ work

$\underline{F}(t, \underline{x}) = -\nabla V(t, \underline{x})$

conservative

$\underline{u} = \nabla f \Longrightarrow \oint_C \underline{u}(\underline{x}) \cdot d\underline{x} = 0$

divergence / Gauss theorem

$\frac{1}{V} \int_{\partial V} \underline{\Gamma} \cdot \underline{n}\, dA = \frac{1}{V} \int_V S_s\, dV \rightarrow \boxed{\int_{\partial V} \underline{\Gamma} \cdot \underline{n}\, dA = \int \nabla \underline{\Gamma}\, dV}$ appeared in 3 out of 4 last exams!

$\nabla \cdot \underline{\Gamma}_s = S_s$

Divergence / Gauss theorem