Lecture

Thursday, 19 December 2019 08:47

$\underline{u} = u^i \underline{a}_i = u_i \underline{a}^i$

$d\underline{x} = \frac{\partial \underline{x}}{\partial x^i} dx^i = \underline{a}_i dx^i$

OCL : $g_{ij} = \begin{pmatrix} h_1^2 & & \\ & h_2^2 & \\ & & h_3^2 \end{pmatrix}$

$\underline{V} = v^j \underline{a}_j \qquad \underline{V} \cdot d\underline{x} = v^j dx^i \, \underline{a}_i \cdot \underline{a}_j = g_{ij} v^j dx^i$

$\underline{V} = v_j \underline{a}^j \qquad \underline{V} \cdot d\underline{x} = v_j dx^i \, \underline{a}^i \cdot \underline{a}_j = v_j dx^i \delta^i_j = v_i dx^i$

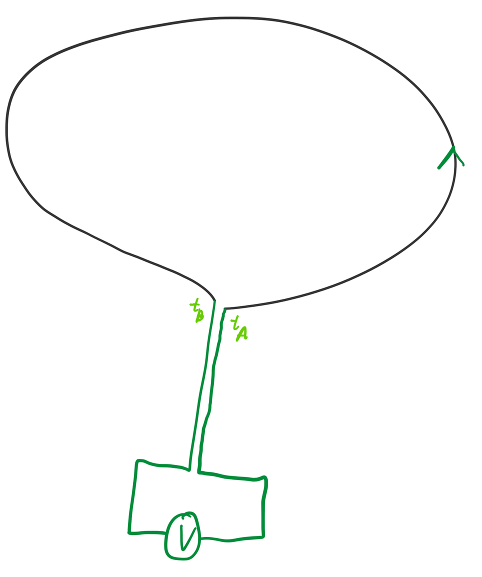
'iff $i=j$

$\underline{V} \cdot d\underline{A}^3 = \sqrt{g} \, \underline{V} \cdot \underline{a}^3 \delta x^1 \delta x^2$

$= \sqrt{g} \, \underbrace{v^i \underline{a}_i}_{\underline{V}} \cdot \underline{a}^3 \delta x^1 \delta x^2$

$= \sqrt{g} \, v^3 \delta x^1 \delta x^2$

gradient	divergence	curl
∇f	$\nabla \cdot \underline{V}$	$\nabla \times \underline{V}$



$EMF = \int_C \underline{E} \cdot d\underline{x} = \int_{\partial A} \underline{E} \cdot d\underline{x} = \mathcal{E} = -\frac{d\phi_m}{dt} = -\frac{d}{dt} \int_A \underline{B} \cdot d\underline{A}$

$$\int_{\partial A} \underline{E} \cdot d\underline{x} = -\frac{d}{dt} \int_A \underline{B} \cdot d\underline{A}$$

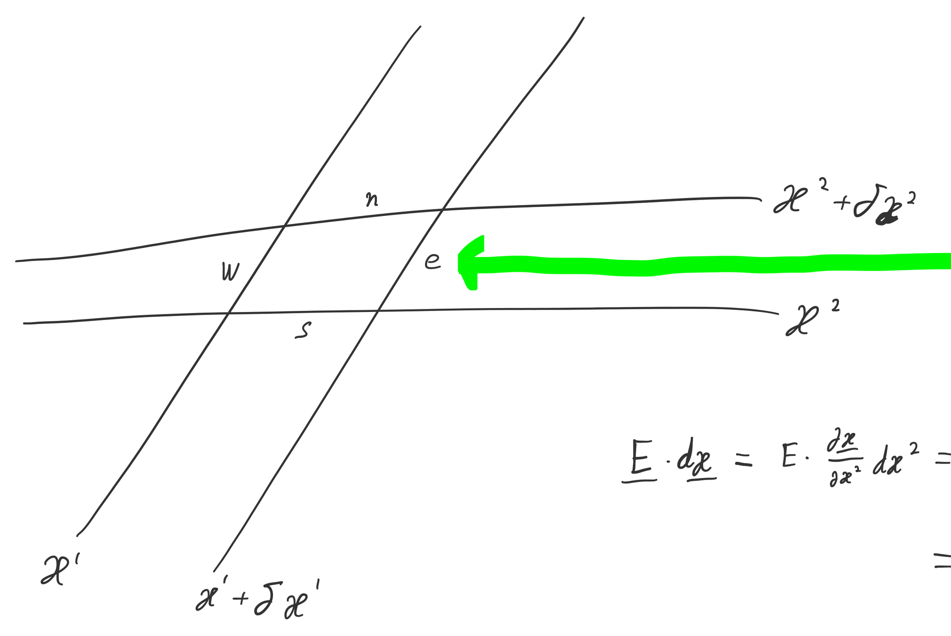
magnetic monopoles do not exist: $\nabla \cdot \underline{B} = 0$

divergence = 0

$\int_V \nabla \cdot \underline{B} = 0 = \int_{\partial V} \underline{B} \cdot d\underline{A} = \phi_m$

$\int \underline{E} \cdot d\underline{x} = -\frac{d}{dt} \int \underline{B} \cdot d\underline{A}$

$\int \nabla \cdot \underline{B} \, dV = 0$



$\underline{E} \cdot d\underline{x} = \underline{E} \cdot \frac{\partial \underline{x}}{\partial x^2} dx^2 = \underline{E} \cdot \underline{a}_2 \, dx^2$

$= E_i \, \underline{a}^i \cdot \underline{a}_2 dx^2 = E_2 \, dx^2$ → e

$E_2 dx^2 |_{east} - E_2 dx^2 |_{west} = \left(E_2 (x^1 + \delta x^1, \dots) - E_2 (x^1, \dots) \right) dx^2$

$= \frac{\partial E_2}{\partial x^1} dx^1 dx^2$

$\int_{\delta A^3} \underline{s} \cdot d\underline{A} = \underline{s} \cdot \delta \underline{A}^3 = \sqrt{g} \, \underline{s} \cdot \underline{a}^3 \delta x^1 \delta x^2$

$= \sqrt{g} \, \underbrace{s^i \underline{a}_i}_{\underline{s}} \cdot \underline{a}^3 \delta x^1 \delta x^2$

$= \sqrt{g} \, s^3 \delta x^1 \delta x^2$

$= \sqrt{g} \, s^3 dx^1 dx^2$

OCL: $\underline{a}_3 = \|a_3\| \; ? \; = h_3 e_{(3)}$

(r, φ, z)

$h_r = 1$
$h_\varphi = r$
$h_z = 1$

$\nabla \times v = \frac{1}{\sqrt{g}} \left(\frac{\partial v_z}{\partial \varphi} - \frac{\partial v_\varphi}{\partial z} \right) \underline{r}$

$+ \frac{1}{r} \left(\frac{\partial v_r}{\partial z} - \frac{\partial v_z}{\partial r} \right) \underline{\varphi}$

$+ \frac{1}{r} \left(\frac{\partial v_\varphi}{\partial r} - \frac{\partial v_r}{\partial \varphi} \right) \underline{z}$

$= \sqrt{\begin{vmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & 1 \end{vmatrix}} = \sqrt{r^2} = r$

$\sqrt{g} = \sqrt{\begin{vmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & 1 \end{vmatrix}} = \sqrt{r^2} = r$

$h_1^2 \quad h_2^2 \quad h_3^2$

$\sqrt{h_1^2 h_2^2 h_3^2} \quad h_1 h_2 h_3$

$\nabla \times E = \frac{1}{r} \left(\frac{\partial E_{(z)}}{\partial \varphi} - \frac{\partial \, r E_{(\varphi)}}{\partial z} \right) e_{(r)}$

$(h_z E_{(z)} = E_z) \quad (h_\varphi E_{(\varphi)} = E_\varphi)$ constant

$+ \frac{1}{r} \left(\qquad \right) e_{(\varphi)}$

$+ \frac{1}{r} \left(\frac{\partial r E_{(\varphi)}}{\partial r} - \frac{\partial E_{(r)}}{\partial \varphi} \right) e_{(z)}$

$(h_\varphi E_{(\varphi)} = E_\varphi)$

$= r \frac{\partial E_{(\varphi)}}{\partial r} + E_{(\varphi)}$

Helmholtz decomposition:

$$\underline{u} = \nabla f + \nabla \times V$$