Exercises Chapter 10

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$\underline{j}$ is defined as total charge-flux out of V

Logically: $-\frac{d}{dt}\int_V \rho_c(\underline{x}, t)\, dV = \int_{\partial V} \underline{j}(\underline{x}, t) \cdot d\underline{A}$

(left side: Charge leaving V during time dt; right side: flow of charge through the surface surrounding V (per unit of time))

From the divergence equation

$$-\frac{d}{dt}\int_V \rho_c(\underline{x}, t)\, dV = \int_V \nabla \cdot \underline{j}(\underline{x}, t)\, dV$$

The volume being fixed gives:

$$\int_V -\frac{d}{dt} \rho_c(\underline{x}, t)\, dV = \int_V \nabla \cdot \underline{j}(\underline{x}, t)\, dV$$

Hence

$$\int_V \left(\frac{d}{dt} \rho_c(\underline{x}, t) + \nabla \cdot \underline{j}(\underline{x}, t)\right) dV = 0$$

Because this holds for any volume (take V very small, then it still holds... (even smaller, etc.)), the integrand is zero:

So, $\frac{d\rho_c}{dt} + \nabla \cdot \underline{j} = 0$

$$E = -\nabla \phi = -\nabla \left(\frac{1}{4\pi\varepsilon_0} \frac{q}{r}\right) = -\frac{\partial}{\partial r}\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r}\right)\underline{e}_r = \frac{q}{4\pi\varepsilon_0 r^2}\, \underline{e}_r \quad \square$$

(where $f = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$, giving $\frac{\partial f}{\partial r}$)

4) $\underline{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \underline{e}_r$

$$\int_{\partial S} \underline{E} \cdot d\underline{A} = \frac{q}{4\pi\varepsilon_0} \int_{\partial S} \frac{1}{r^2} \underline{e}_r \cdot \underline{e}_r\, dA = \frac{4\pi r^2}{4\pi\varepsilon_0} \frac{q}{r^2} = \frac{q}{\varepsilon_0}$$

5) Taking the divergence on both sides of equation 10.16 gives:

$$\mu_0^{-1}\, \nabla \cdot (\nabla \times B) = \nabla \cdot \underline{j}$$

by formula sheet eq (25),

$$0 = \nabla \cdot \underline{j}$$

Which is inconsistent with $\frac{\partial \rho_c}{\partial t} + \nabla \cdot \underline{j} = 0$ if $\frac{\partial \rho_c}{\partial t} \neq 0$, which, in general, is the case.

Again taking the divergence,

$$\mu_0^{-1}\, \nabla \cdot (\nabla \times B) = \nabla \cdot \underline{j} + \varepsilon_0\, \nabla \cdot \frac{\partial E}{\partial t}$$

$$0 = \nabla \cdot \underline{j} + \varepsilon_0 \frac{\partial}{\partial t} \frac{\rho_c(\underline{x})}{\varepsilon_0}$$

$0 = \nabla \cdot \underline{j} + \frac{\partial \rho_c(\underline{x})}{\partial t}$, which is consistent with equation