Exercises Chapter 11

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Consider the field $\underline{v} = \underline{e}_x$ in Cartesian coordinates, which can have vector potential

$$\underline{A} = y\underline{e}_z \quad \text{, but also}$$

$$\underline{A} = y\underline{e}_z + x\underline{e}_x$$

In general, one can add $\nabla\psi$ to the field, as

$$\nabla \times (\nabla \psi) = \underline{0}$$

2) Taking the divergence of $\underline{v}$ should result in 0.

$$\nabla \cdot (\nabla f \times \nabla g) =$$

$$(\nabla \times (\nabla f)) \cdot \nabla g - \nabla f \cdot (\nabla \times (\nabla g))$$

$$\underline{0} \cdot \nabla g - \nabla f \cdot \underline{0} = 0$$

$\underline{A}$ is a vector potential of $\underline{v}$, because

$$\nabla \times \underline{A} = \nabla \times (f \nabla g)$$

$$= f(\nabla \times (\nabla g)) + (\nabla f) \times \nabla g$$

$$= f(\underline{0}) + \nabla f \times \nabla g$$

$$= \nabla f \times \nabla g = \underline{v}$$

3) Taking the divergence of $\underline{v}$:

$$\nabla \cdot \underline{v} = \nabla \cdot (\underline{a} \times \underline{x})$$

$$= (\nabla \times \underline{a}) \cdot \underline{x} - \underline{a} \cdot (\nabla \times \underline{x})$$

$$= (\nabla \times \underline{a}) \cdot \underline{x} - \underline{a} \cdot \underline{0}$$

$$= (\nabla \times \underline{a}) \cdot \underline{x}$$

$$= 0 \text{, so the field is solenoidal}$$

$$\underline{A} = f(\underline{x})\underline{x}$$

then $\nabla \times \underline{A} = \nabla \times (f(\underline{x})\underline{x})$

$$= \nabla \times (f\underline{x})$$

$$= f(\nabla \times \underline{x}) + (\nabla f) \times \underline{x}$$

$$= (\nabla f) \times \underline{x} \text{, which is equal to } \underline{v} \text{ if } \nabla f = \underline{a}$$